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Isocurvature Fluctuations
in Tracker Quintessence Models

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Abstract

We consider effects of the isocurvature perturbation in the framework of the tracker-type quintessence models. During the inflation, fluctuations in the amplitude of the quintessence field are generated, which provide isocurvature component of the cosmic density perturbation. Contrary to the conventional notion, it is shown that effects of the isocurvature fluctuation may become sizable in some cases, and in particular, the cosmic microwave background angular power spectrum may be significantly enhanced due to the effect of the isocurvature mode. Such an enhancement may be detectable in on-going and future experiments.

Recent cosmological observations suggest that there exists a dark energy which must be added to the matter density in order to reach the critical density [1]. Although the cosmological constant is usually assumed as the dark energy, in the past years, a slowly evolving scalar field, dubbed as "quintessence" has been proposed as the dark energy [2]. There are some differences between the cosmological constant and quintessence. Firstly, for the quintessence, the equation-of-state parameter $\omega_Q = \rho_Q/p_Q$ varies with time, whilst for the cosmological constant, it remains a fixed value $\omega_\Lambda = -1$. Secondly, since the quintessence is a scalar field, it fluctuates.

One of the observational discriminator of these models is the cosmic microwave background (CMB) anisotropies. Many authors have studied the effects of the quintessence field on the CMB anisotropy in various models. As mentioned above, the quintessence field fluctuates, so its fluctuations should be taken into account in calculating the CMB anisotropy. Generally, the initial fluctuations of the quintessence fields are generated during inflation. Since these fluctuations behave as the isocurvature mode, there can exist both adiabatic and isocurvature perturbations. Implications of the isocurvature modes have been studied in [3], and in particular for the cosine-type quintessence model, it was pointed out that the isocurvature fluctuations may be detectable in on-going and future experiments by the present authors [4].

In this letter, we focus on the effects of the isocurvature fluctuations on the CMB anisotropy in the tracker-type quintessence models. The tracker-type quintessence models have attractor-like solutions which alleviate the initial condition problems. When we take initial conditions such that the quintessence field starts to roll down the potential at early epoch, damping effect on fluctuation of the quintessence field is significant. So, in this case, even if we take a nonzero value of primordial quintessence-field fluctuation, the fluctuation goes to zero because of the damping effects. But we will show that, in some cases, the damping effect is not so significant. In such cases, the effect of the isocurvature fluctuations may be sizable and affect the CMB angular power spectrum.

Since the effects of the isocurvature fluctuations depend on models of quintessence, first, we classify the tracker-type quintessence models. Although many models have been proposed [2, 3, 5–20], they can be classified into two groups, using the evolution of the energy density of the quintessence ρ_Q. The evolution of the energy density of the tracker-type models can be written in a simple form. Tracker-type quintessence models, as previously mentioned, have an attractor-like solution in a sense that a very wide range of initial conditions converge to a common evolutionary track. We call the epoch when this attractor-like solution is realized as "tracking regime." In the tracking regime, ω_Q is almost constant and $1 - \omega_Q^2$ is significantly different from zero [21]. This means that the kinetic and potential energy of the quintessence have a fixed ratio. Thus, the relation $\dot{Q}^2 \propto V(Q)$ holds in the tracking regime. When this relation is satisfied, we can show that the energy density of the quintessence is proportional to a^{-n}, where n is a constant. On the contrary, the energy density of the dominant component of the universe is written as $\rho_D \propto a^{-m}$ (for the matter and radiation dominations, $m = 3$ and 4, respectively). When $n = m$, the energy density of the quintessence can be a significant fraction of the total energy density of the

universe from earlier epoch. On the contrary, when $n < m$, ρ_Q cannot be significant at early epoch.[#1] For a purpose of this letter, we will consider two models as representative models; "AS model" [6] and "Ratra-Peebles model" [13] for models with $n = m$ and $n < m$, respectively.

Before we show the evolution of the quintessence energy density from numerical calculations, we investigate it analytically. To study the evolution of the quintessence fields, we write down the basic equations. In a spatially flat homogeneous universe which contains a perfect fluid with energy ρ and pressure p and a quintessence field Q with energy ρ_Q and pressure p_Q, the Friedmann equation becomes

$$H^2 = \frac{1}{3M_*^2}(\rho + \rho_Q), \tag{1}$$

where H is the Hubble parameter and $M_* \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale. The energy density and pressure of the quintessence field are

$$\rho_Q = \frac{1}{2}\dot{Q}^2 + V(Q), \qquad p_Q = \frac{1}{2}\dot{Q}^2 - V(Q), \tag{2}$$

where the dot represents the derivative with respect to time t. The quintessence field Q obeys the equation of motion

$$\ddot{Q} + 3H\dot{Q} + \frac{dV}{dQ} = 0. \tag{3}$$

The energy conservation equation for each component is, in case where there is no energy exchange

$$\dot{\rho} = -3H(\rho + p). \tag{4}$$

Thus, when the universe is dominated by a perfect fluid with the energy density $\rho_D \propto a^{-m}$, the scale factor behaves as

$$a \propto t^{2/m}. \tag{5}$$

In this case, from Eqs.(2) and (3),

$$\ddot{Q} + \frac{6}{mt}\dot{Q} + \frac{dV}{dQ} = 0, \tag{6}$$

First, let us discuss the evolution of ρ_Q for the AS model which has the potential of the form

$$V(Q) = \left[(Q-b)^2 + a\right]e^{-\lambda Q} = f(Q)e^{-\lambda Q}, \tag{7}$$

[#1] The case with $n > m$ is also possible [22], but almost all model which have been proposed so far, can be categorized into the type-I or the type-II models. So we will not consider the case with $n > m$ in this letter.

where a, b and λ are model parameters. Since $f(Q)$ changes more slowly than $e^{-\lambda Q}$ in the tracking regime, in the analytic investigations below, we approximately treat $f(Q)$ as a constant. Substituting Eq.(7) into Eq.(6), we obtain

$$\ddot{Q} + \frac{6}{mt}\dot{Q} - f(Q)\lambda e^{-\lambda Q} = 0. \tag{8}$$

The solution to this equation is $Q = A \ln \lambda Bt$, where A and B are constants. In the tracking regime, $\dot{Q}^2$ is proportional to $V(Q)$, thus $\rho_Q \propto t^{-2}$ in this model. Using Eq.(5), it is shown that

$$\rho_Q \propto a^{-m}. \tag{9}$$

Therefore, in the AS model, the energy density of the quintessence has the same a dependence as that of the dominant component of the universe. This means that the energy density of the quintessence can be a sizable fraction of the total energy density of the universe at early epoch. In fact, the above result does not depend on the detailed structure of the function $f(Q)$ as far as $f(Q)$ is a slowly varying function on the tracking regime. Thus, the same discussion is applicable to other types of quintessence models with $V(Q) = f(Q)e^{-\lambda Q}$.

Next, we consider the Ratra-Peebles model which has the potential of the form,

$$V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha}, \tag{10}$$

where Λ and $\alpha(> 0)$ are model parameters. With this potential, the equation of motion of the quintessence field becomes

$$\ddot{Q} + \frac{6}{mt}\dot{Q} - \alpha\frac{\Lambda^{4+\alpha}}{Q^{\alpha+1}} = 0. \tag{11}$$

This equation has the solution $Q = Ct^\nu$ where $\nu = 2/(2+\alpha)$ and C is a constant. Since α is taken to be positive, the value of ν is $0 < \nu < 1$. The energy density of the quintessence in this model becomes

$$\rho_Q \propto a^{-m(1-\nu)}. \tag{12}$$

Thus, in the Ratra-Peebles model, ρ_Q decreases more slowly than that of the dominant component of the universe, which implies that the energy density of the quintessence cannot be a significant fraction of the energy density in the early universe.

Now we show evolutions of the energy density of the quintessence models from the numerical calculations. In Fig.1, we show the evolution of the quintessence energy density for the AS model, where we take several values of the initial amplitude for the quintessence field. From the figure, we can read off when the quintessence field starts to enter the tracking regime for several values of the initial amplitudes. We can also see the property of

the AS model; the energy density of the quintessence field can have $O(0.1)$ contributions to the total energy density of the universe from early epoch.

In Fig. 2, the evolution of the energy density are shown for the Ratra-Peebles model. Different from the AS model, the quintessence energy density of this type of model is negligible for $z \gg O(1)$. This fact has an important implication to the effects of the isocurvature fluctuations on the CMB angular power spectrum.

Next let us discuss the effects of the isocurvature fluctuations on the CMB angular power spectrum. For this purpose, we study the evolutions of the fluctuations of the quintessence analytically. We decompose the quintessence field Q as

$$Q(t, \vec{x}) = \bar{Q}(t) + q(t, \vec{x}), \tag{13}$$

where q is the perturbation of the amplitude of the quintessence field. The equation of motion for q is, in the conformal Newtonian gauge,

$$\ddot{q} + 3H\dot{q} - \left(\frac{a}{a_0}\right)^{-2} \partial_i^2 q + \frac{d^2V(\bar{Q})}{dQ^2} q = (\dot{\Psi} - 3\dot{\Phi})\dot{\bar{Q}} - 2\left(\frac{a}{a_0}\right)^2 \frac{dV}{dQ}\Psi, \tag{14}$$

where the perturbed line element in the conformal Newtonian gauge is given by

$$ds^2 = -(1 + 2\Psi)dt^2 + \left(\frac{a}{a_0}\right)^2 (1 + 2\Phi)\delta_{ij}dx^i dx^j$$

$$= \left(\frac{a}{a_0}\right)^2 \left[-(1 + 2\Psi)d\tau^2 + (1 + 2\Phi)\delta_{ij}dx^i dx^j\right], \tag{15}$$

where a is the scale factor at time t, a_0 the scale factor at the present time and τ is the conformal time.

From Eq.(14), we can show the damping behavior of the quintessence field in the tracking regime. To solve this equation, we write down the second derivative of the potential as a function of the sound speed c_s^2 of a quintessence field Q [5]

$$\frac{d^2V(\bar{Q})}{dQ^2} = \frac{3}{2}H\dot{c}_s^2 + \frac{3}{2}H^2(c_s^2 - 1)\left[\frac{\dot{H}}{H^2} - \frac{3}{2}(c_s^2 + 1)\right], \tag{16}$$

where the sound speed of the quintessence field is written as

$$c_s^2 = \frac{\dot{p}_Q}{\dot{\rho}_Q} = \frac{\ddot{\bar{Q}} - dV/dQ}{\ddot{\bar{Q}} + dV/dQ}. \tag{17}$$

Since the kinetic energy is proportional to the potential energy in the tracking regime, the relation $\ddot{\bar{Q}} \propto dV/dQ$ holds. So the sound speed of the quintessence field c_s^2 is a constant during this regime. Therefore $\dot{c}_s^2$ can be set to zero in Eq.(16). When the quintessence is a subdominant component of the universe, the Hubble parameter H can be written using the equation-of-state parameter $\omega_D = p_D/\rho_D$ of the dominant component of the universe,

$$H = \frac{2}{3(1 + \omega_D)t}. \tag{18}$$

To study the isocurvature mode at the superhorizon scale, we neglect the k dependence and the right hand side of Eq.(14). Then, using the above relations, the equation of motions can be written as

$$\ddot{q} + \frac{2}{(1+\omega_D)t}\dot{q} + \frac{1}{(1+\omega_D)^2 t^2}(1-c_s^2)(c_s^2+\omega_D+2)q = 0. \tag{19}$$

This equation has power-law solutions like $q \propto t^\xi$, where the power-law index ξ is

$$\xi = \frac{\omega_D - 1}{2(\omega_D + 1)}\left[1 \pm \sqrt{1 - \frac{4}{(\omega_D - 1)^2}(1-c_s^2)(c_s^2+\omega_D+2)}\right]. \tag{20}$$

Since c_s^2 is equal to ω_Q in the tracking regime [5] and $\omega_{D,Q}$ cannot be larger than unity, the real part of ξ is always negative. Therefore q damps with time. It follows that if we take $q \neq 0$ initially, q damps to zero in case that the quintessence field experiences long period of tracking [3,5]. But, in the case that the quintessence field enters the tracking regime at later time, the fluctuations may not damp so much. In this case, the isocurvature fluctuations (i.e., $q \neq 0$) can affect CMB anisotropies.

The primordial fluctuations in the quintessence amplitude is generated in the early universe, probably during the inflation. When the mass of the quintessence field is negligible compared to Hubble parameter during inflation H_{inf}, the primordial fluctuation is [23][#2]

$$q_{\text{inf}}(k) = \frac{H_{\text{inf}}}{2\pi}. \tag{21}$$

To parameterize the size of the isocurvature contribution, we define the following quantity r_q:

$$r_q \equiv \frac{q_{\text{RD}}}{M_* \Psi_{\text{RD}}^{(\text{adi})}}, \tag{22}$$

where q_{RD} is the fluctuation of the quintessence field in the deep radiation-dominated epoch when the quintessence field is slow-rolling, while $\Psi_{\text{RD}}^{(\text{adi})}$ is Ψ from the adiabatic mode (i.e., contribution from the inflaton-field fluctuation) in the deep radiation-dominated epoch. For a given model of slow-roll inflation, $\Psi_{\text{RD}}^{(\text{adi})}$ is [24]

$$\Psi_{\text{RD}}^{(\text{adi})} = \frac{4}{9}\left(\frac{H_{\text{inf}}}{2\pi}\frac{V_{\text{inf}}}{M_*^2 V_{\text{inf}}'}\right), \tag{23}$$

where V_{inf} is the inflaton potential and V_{inf}' is its derivative with respect to the inflaton field χ.

[#2] If $m_q \gtrsim H_{\text{inf}}$, the primordial fluctuation damps to zero rapidly and cannot affect the CMB anisotropies. So we do not consider such a case.

The parameter r_q depends on the detailed scenario of cosmology. In a simple case with $q_{\mathrm{RD}} = q_{\mathrm{inf}}$,

$$r_q \simeq \frac{9}{4} \frac{M_* V'_{\mathrm{inf}}}{V_{\mathrm{inf}}}. \tag{24}$$

For example, for the chaotic inflation with $V_{\mathrm{inf}} \propto \chi^p$ where p is an integer, r_q is given by

$$r_q|_{\mathrm{chaotic}} = \frac{9 p M_*}{4 \chi(k_{\mathrm{COBE}})}, \tag{25}$$

where $\chi(k_{\mathrm{COBE}})$ represents the inflaton amplitude at the time when the COBE scale crosses the horizon. Numerically, we found $r_q \simeq 0.3 - 0.6$ for $p = 2 - 10$. The ratio r_q may become larger, however, in more complicated cases. For example, if the kinetic function for the quintessence field Z_Q varies during inflation, q also changes. This may happen if Z_Q depends on the inflaton field. Therefore, in general the ratio r_q is model dependent and we treat r_q as a free parameter in our analysis.

Now we show the results from numerical calculations. We used the modified version of CMBFAST [25] to calculate the CMB angular power spectrum C_l which is defined as

$$\langle \Delta T(\vec{x}, \vec{\gamma}) \Delta T(\vec{x}, \vec{\gamma}') \rangle = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\vec{\gamma} \cdot \vec{\gamma}'), \tag{26}$$

where $\Delta T(\vec{x}, \vec{\gamma})$ is the temperature fluctuation of the CMB pointing to the direction $\vec{\gamma}$, and P_l is the Legendre polynomial. The average is over the position $\vec{x}$.

In Fig. 3, we plot the CMB angular power spectrum C_l in the AS model with several values of the initial amplitude Q_{in} and the ratio r_q.[#3] For comparison, we also plotted C_l in the cosmological constant case.

First, we discuss the adiabatic case (i.e., $r_q = 0$), in particular, focusing on differences between the quintessence and the cosmological constant cases. One can see that the locations of the acoustic peaks may differ in two cases. In our case, locations of the peaks mostly depend on the angular diameter distance to the last scattering surface (LSS) $r_\theta(\tau_*)$, and the location of the n-th peak in the l space is estimated as [26, 27]

$$l_n \simeq \frac{r_\theta(\tau_*)}{r_s(\tau_*)} n\pi, \tag{27}$$

where $r_s(\tau_*)$ is the sound horizon at the recombination. The angular diameter distance becomes smaller when the energy density of the universe is large since the expansion rate becomes larger in this case. The energy density of the universe in the earlier epoch becomes larger in the case with the quintessence. Thus, in the quintessence case, the locations of the acoustic peaks are shifted to lower multipoles due to this effect. Such a shift is observed in Fig. 3.

[#3] With the cosmological and model parameters used in Fig. 3, we checked that theoretical predictions of C_l are in good agreements with current observational data with proper normalization.

Another point we should mention is the height of acoustic peaks (relative to C_l at lower multipoles). In the AS model, the driving effect is important to understand the height of the peaks; in the radiation-dominated epoch, the amplitudes of the acoustic oscillation is boosted just after the horizon crossing due to the decay of Ψ. Compared to the cosmological constant case, the expansion rate of the universe becomes larger in the quintessence case, resulting in faster decay of the gravitational potential Ψ. This effect enhances the driving effect. Thus, in particular in the AS models, height of the acoustic peaks are enhanced relative to lower multipoles [6].

Next we consider the case with $r_q \neq 0$. For this purpose, it is instructive to study the behavior of the perturbations generated by the fluctuation of the quintessence amplitude. Evolutions of the perturbations are governed by the following equations; for the metric perturbation,

$$k^2 \Phi = 4\pi G \left(\frac{a}{a_0}\right)^2 \rho_{\text{tot}} \left[\delta_{\text{tot}} + 3\mathcal{H}(1 + \omega_{\text{tot}})\frac{V_{\text{tot}}}{k}\right], \tag{28}$$

and for the photon, at the superhorizon scale,

$$\delta'_\gamma = -\frac{3}{4}kV_\gamma - 4\Phi', \quad V'_\gamma = k\left(\frac{1}{4}\delta_\gamma + \Psi\right), \tag{29}$$

where $\mathcal{H} \equiv a'/a$, δ_γ and V_γ (δ_{tot} and V_{tot}) are density perturbation and velocity perturbation of photon (those of total matter density) which are defined in the Newtonian gauge, and the prime represents the derivative with respect to the conformal time τ. In the deep radiation-dominated epoch when the quintessence field is slow-rolling, the fluctuations in energy density, pressure and velocity of the quintessence are approximated as

$$\delta\rho_Q \simeq -\delta p_Q \simeq \frac{dV}{dQ}q_{\text{RD}}, \quad (\rho_Q + p_Q)V_Q = k\left(\frac{a}{a_0}\right)^{-2} Q' q_{\text{RD}}. \tag{30}$$

In the deep radiation-dominated epoch when the scale we concern is out of horizon, we can study the behavior of the perturbations by expanding each variables with the conformal time τ. Neglecting the shear stress from neutrino (i.e., using $\Pi_{\text{tot}} = 0$), $\Psi \simeq -\Phi$. Thus we find

$$\Psi \simeq Cq_{\text{RD}}\tau^4, \quad \delta_\gamma \simeq 4Cq_{\text{RD}}\tau^4, \quad V_\gamma \simeq \frac{2}{5}Ckq_{\text{RD}}\tau^5, \tag{31}$$

with

$$C = -\frac{1}{126}\frac{dV}{dQ}\frac{1}{M_*^4}\rho_{\text{rad}0}, \tag{32}$$

where $\rho_{\text{rad}0}$ is the energy density of radiation at the present epoch. Here, we keep the leading term in τ. In the radiation-dominated epoch, $\tau \propto t^{1/2}$ and all of these variables (i.e., Ψ, δ_γ and V_γ) grow with time for the isocurvature mode. In addition, we can see that,

at the superhorizon scale, Ψ has the same k dependence as q_{RD}. Thus, if q_{RD} has no scale dependence, Ψ is scale-invariant.

This behavior can be confirmed by numerical calculations. In Fig. 4, we show evolution of the gravitational potential generated by the fluctuation in the quintessence amplitude. When the quintessence is in the slow-roll regime, Ψ is proportional to a^4. (Notice that, in the radiation-dominated epoch, $a \propto \tau$.) At the time when the quintessence field enters the tracking regime, Ψ ceases to grow since q starts to damp. Thus, Ψ has its maximum value at this epoch. Consequently, the CMB angular power spectrum is the most enhanced at the angular scale which enters the horizon at the time when the tracking regime starts.

In the AS model, since Ω_Q can be $O(0.1)$ in the tracking regime, fluctuation of the quintessence field may significantly affect the angular power spectrum C_l. To see the effect, in Fig. 3, we plot the CMB angular power spectrum for the case with $r_q = 5$. (Notice that the enhancement due to the isocurvature contribution, $C_l(r_q \neq 0)/C_l(r_q = 0) - 1$, is proportional to r_q^2.) As we can see from the figure, when $r_q \neq 0$, the angular power spectrum C_l is enhanced.

To study the enhancement in C_l by the isocurvature fluctuation, we plot the ratio $C_l(r_q = 5)/C_l(r_q = 0)$ with several values of the initial amplitude in Fig. 5. We can clearly see that the multipole with the largest enhancement depends on the initial amplitudes of the quintessence fields. As we discussed before, this angular scale is determined by the time when the tracking regime starts; the effect of the isocurvature fluctuation is not significant for the angular scale much larger than and much smaller than this scale because of the damping effect of q or smallness of Ω_Q. As we can see, the later the quintessence field enters the tracking regime, C_l is most enhanced at lower multipole l. Since the epoch when the quintessence field enters the tracking regime depends on its initial amplitude, the enhancement in C_l may give information about the initial condition of the quintessence fields. In addition, with $r_q = 5$, the enhancement in C_l can be as large as 50 % relative to the case with $r_q = 0$ when $Q_{\mathrm{in}} = 121 M_*$, which can be detectable in on-going and future satellite experiments [28, 29]. Thus studying effects of isocurvature fluctuations provides us implications on model building of the quintessence, which is complementary to studying the adiabatic fluctuation.

Finally, we comment on the case with the Ratra-Peebles model. We also calculated the CMB angular power spectrum in the Ratra-Peebles model. However, in this model, effects of isocurvature fluctuation are very small since Ω_Q is very small in the early universe. For example, even if we take $r_q = 5$ with the model parameters $\Lambda = 5.122 \times 10^6$ GeV and $\alpha = 6$, enhancement in C_l is less than a few %.

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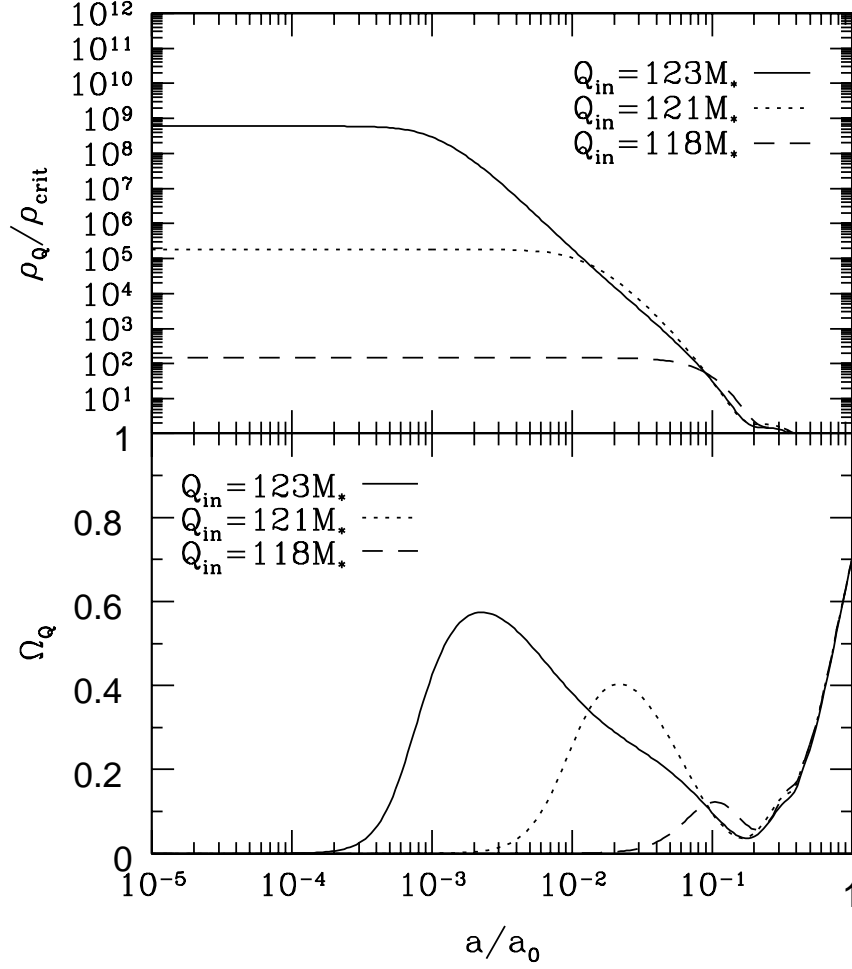[29] PLANCK home page (http://astro.estec.esa.nl/SA-general/Projects/Planck).

Figure 1: Evolution of the energy density in the AS model. The model parameters are taken to be $\lambda = 2.2/M_*$, $A = 0.01M_*^2$ and $B = 123.6805M_*$. The initial amplitude are $Q_{in} = 123M_*$ (solid line), $121M_*$ (dotted line), and $118M_*$ (dashed line). The cosmological parameters are taken to be $\Omega_0 = 0.3$, $\Omega_b h^2 = 0.019$ and $h = 0.65$, where h is the Hubble parameter in units of 100 km/s/Mpc.
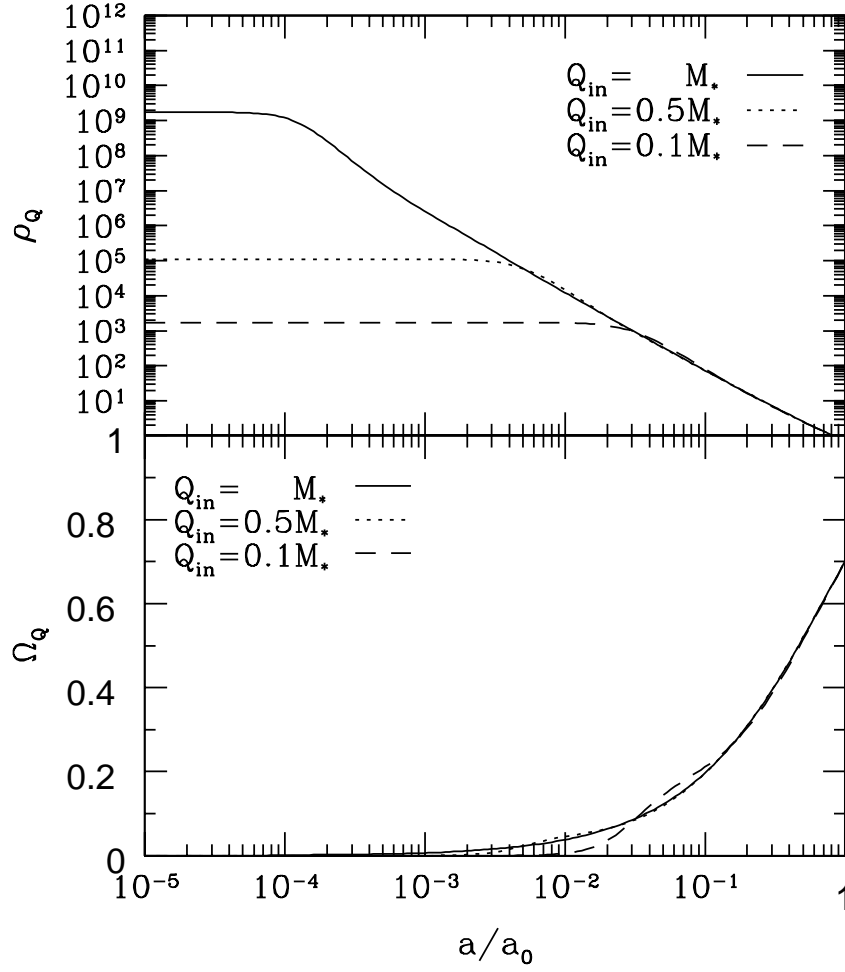
Figure 2: Evolution of the energy density in the Ratra-Peebles model. The parameter we take here, $\Lambda = 5.122 \times 10^6$ GeV and $\alpha = 6$. The initial amplitudes of the quintessence field are $Q_{\text{in}} = M_*$ (solid line), $0.5M_*$ (dotted line) $0.1M_*$ (dashed line). The cosmological parameters are the same as those in Fig.1.
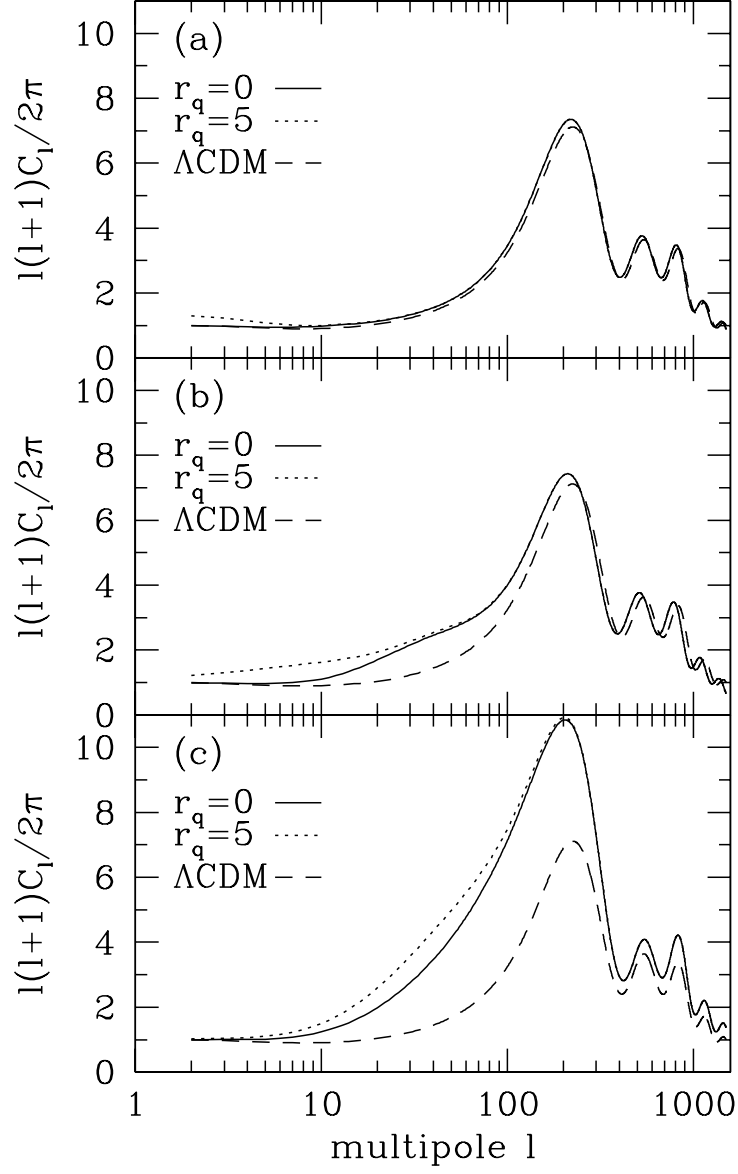
Figure 3: The CMB angular power spectrum in the AS model. The initial amplitudes are (a) $Q_{in} = 123M_*$, (b) $Q_{in} = 121M_*$, and (c) $Q_{in} = 118M_*$. The cosmological and model parameters are the same as those in Fig. 1. The overall normalization is arbitrary in this figure.
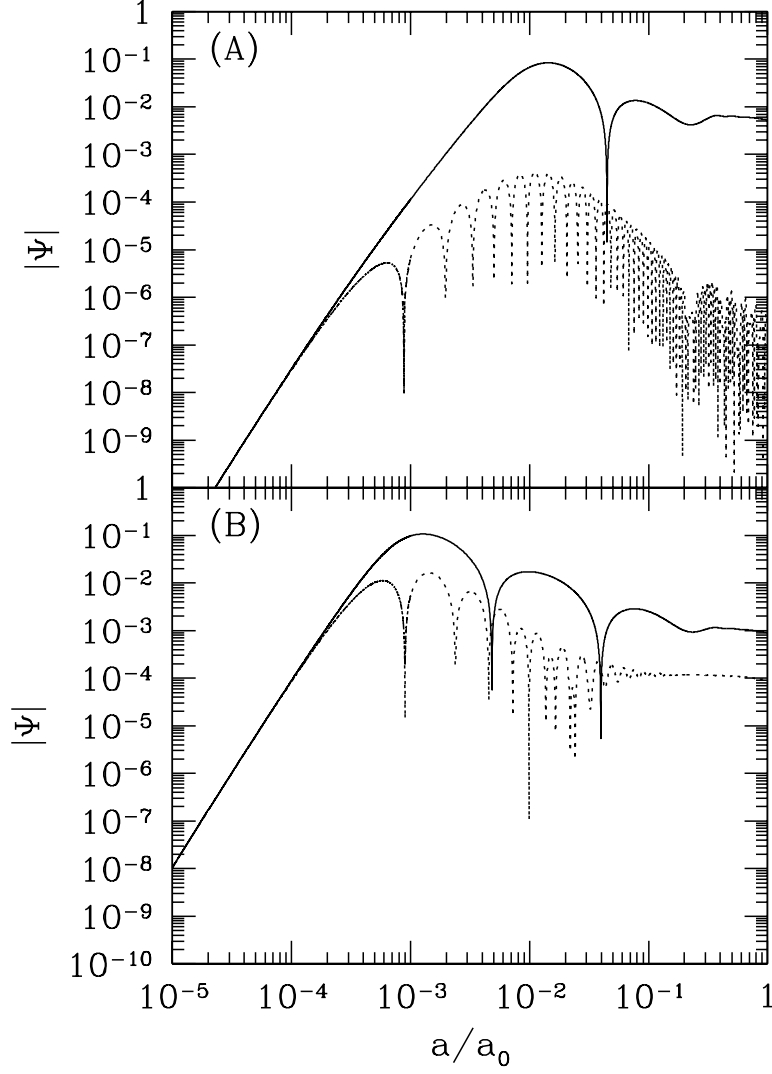
Figure 4: The evolutions of $|\Psi|$ of purely isocurvature mode with (A) $Q_{\text{in}} = 121M_*$ and (B) $Q_{\text{in}} = 118M_*$. We take $k = 1 \times 10^{-4}h$ Mpc^{-1} (solid line) and $k = 1 \times 10^{-2}h$ Mpc^{-1} (dotted line). The model and cosmological parameters are the same as those in Fig. 1.
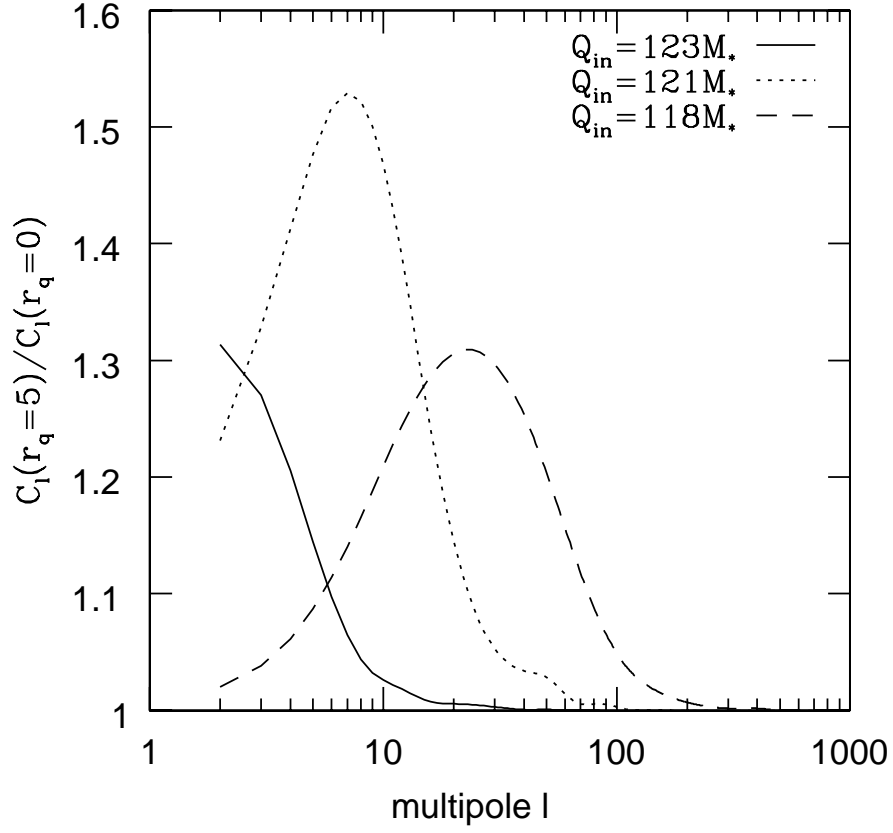
Figure 5: The ratio $C_l(r_q = 5)/C_l(r_q = 0)$ in the AS model. The initial amplitude are taken to be $123M_*$ (solid line), $121M_*$ (dotted line), and $118M_*$ (dashed line). The cosmological and model parameters are the same as those in Fig. 1. Notice that the quantity $C_l(r_q \neq 0)/C_l(r_q = 0) - 1$ is proportional to r_q^2.